Limits Review

Name ______________________

Use an algebraic simplification to help find the limit, if it exists.

1. $\lim\limits_{x\to -1}\frac{x-3}{x+4} = \frac{-1-3}{-1+4} = -\frac{4}{3}$

2. $\lim\limits_{x\to 2}\frac{\sqrt{x}-\sqrt{2}}{x-2} = \lim\limits_{x\to 2}\frac{(\cancel{\sqrt{x}-2})}{(\cancel{\sqrt{x}-\sqrt{2}})(\sqrt{x}+\sqrt{2})} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

3. $\lim\limits_{x\to 9}\frac{x-9}{3-\sqrt{x}} = \lim\limits_{x\to 9}\frac{(\cancel{\sqrt{x}-3})(\sqrt{x}+3)}{-(\cancel{\sqrt{x}-3})} = \lim\limits_{x\to 9}\left(-(\sqrt{x}+3)\right) = -(\sqrt{9}+3) = -6$

4. $\lim\limits_{x\to -2}\sqrt{8+x^3} = \sqrt{8-8} = 0$

5. $\lim\limits_{h\to 0}\frac{(x+h)^2-x^2}{h} = \lim\limits_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h} = \lim\limits_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim\limits_{h\to 0}(2x+h) = 2x$

6. $\lim\limits_{x\to\infty}\frac{2x^2-x+3}{x^3+1} = 0$

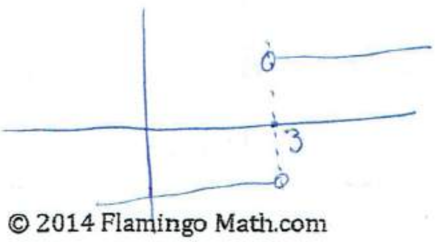
7. $\lim\limits_{x\to 1}\frac{x^2-x}{2x^2+5x-7} = \lim\limits_{x\to 1}\frac{x(\cancel{x-1})}{(2x+7)(\cancel{x-1})} = \lim\limits_{x\to 1}\frac{x}{2x+7} = \frac{1}{2+7} = \frac{1}{9}$

8. $\lim\limits_{x\to\infty}\left(\frac{1}{x}-\frac{x}{x-1}\right) = \lim\limits_{x\to\infty}\frac{1}{x} - \lim\frac{x}{x-1} = 0-1 = -1$

9. $\lim\limits_{x\to 2}\frac{x^2-x-2}{(x-2)^2} = \lim\limits_{x\to 2}\frac{(\cancel{x-2})(x+1)}{(\cancel{x-2})(x-2)} = \lim\limits_{x\to 2}\frac{x+1}{x-2} = \text{DNE}$

10. $\lim\limits_{x\to -\infty}\frac{-x^3+5x}{3x^2-2} = +\infty$

11. $\lim\limits_{x\to 3^+}\frac{|x-3|}{x-3} = 1$



12. $\lim\limits_{x\to\infty}\frac{x^{1/3}}{x^3+1} = 0$

13. If $f(x) = \frac{1}{x-2}$ and $\lim_{x \to (-k+1)} f(x)$ *does not exist*, then k must be ________.

14. Let f be the function defined by $f(x) = \begin{cases} x^2 - 3, & x > -1 \\ 1 - x, & x \leq -1 \end{cases}$

A. Find $f(-1)$ $= 1-(-1) = 2$

B. Find $\lim_{x \to -1^+} f(x)$ $= (-1)^2 - 3 = -2$

C. Find $\lim_{x \to -1^-} f(x)$ $= 1-(-1) = 2$

D. Explain why f is not continuous at $x = -1$ $\lim_{x \to -1^+} f(x) \neq \lim_{x \to -1^-} f(x) \Rightarrow \lim_{x \to -1} f(x) = [illegible]$

E. Give an expression to use instead of $(1 - x)$ that would guarantee f is a continuous function.

15. Let f be the function defined by $f(x) = \frac{x-1}{x^2-3x+2}$

A. List all values of x where the function is continuous.

B. At what value(s) of x is the function discontinuous

Domain $\frac{x-1}{x^2-3x+2}$ $\frac{x-1}{(x-2)(x-1)}$

$x \neq 2$

$x \neq 1$

continuous $(-\infty, 1) \cup (1, 2) \cup (2, +\infty)$

discontinuous at $x = 1$ and $x = 2$

ICM Test Review: Limits and Continuity

1. Find the equation of the line tangent to the curve $y = x^2 - 2x + 1$ at (2, 1).

$x_0 = 2$

$f(2+h) = (2+h)^2 - 2(2+h) + 1 =$
$= 2h + h^2 + 1$

$f(2) = (2)^2 - 2\cdot(2) + 1 = 1$

$m = \lim_{h\to 0} \frac{2h + h^2 + 1 - 1}{h} = \lim_{h\to 0} \frac{\cancel{h}(2+h)}{\cancel{h}} =$

$y = y_0 + m(x - x_0)$ $\quad y_0 = f(2) = 1$

$y = 1 + 2(x-2)$ $\quad \boxed{y = 2x - 3}$

2. Consider the function $y = f(x)$ shown below. Give the value of each limit.

a) $\lim_{x\to a} f(x)$

b) $\lim_{x\to b^-} f(x)$

c) $\lim_{x\to c} f(x)$

d) $\lim_{x\to d} f(x)$

3. For what values of *x* is each of the following functions discontinuous?

a) $f(x) = \frac{x}{x+3}$ $\quad x = -3$

b) $f(x) = \frac{x+2}{x^2+x-2} = \frac{x+2}{(x+2)(x-1)}$ $\quad x = -2$, $x = 1$

c) $f(x) = \begin{cases} -x & ,x < -2 \\ \frac{1}{2}x + 2 & ,-2 \le x \le 0 \\ \sqrt{4-x^2} & ,0 \le x \le 2 \\ -x & ,x > 2 \end{cases}$

d) $f(x) = \frac{|x^2-16|}{x^2-16} = \frac{|x^2-16|}{(x-4)(x+4)}$

$x = -4$ and $x = 4$

e) $f(x) = \frac{1}{x^2-16} = \frac{1}{(x-4)(x+4)}$

$x = -4$ and $x = 4$

f) $f(x) = \frac{x^2-x-2}{x^2-2x} = \frac{(x-2)(x+1)}{x(x-2)}$

$x = 0$ and $x = 2$

g) $f(x) = \frac{x+2}{x^3-8} = \frac{x+2}{(x-2)(x^2+2x+4)}$

$x = 2$

4. If $f(x) = \frac{1}{5 - \sqrt{x^2+16}}$ is not continuous at c, then c = $\underline{-3, 3}$.

$5 - \sqrt{x^2+16} = 0$

$\sqrt{x^2+16} = 5$

$x^2 + 16 = 25$

$x^2 = 9$

$x = -3$

$x = 3$

5. Find the limit, if it exists.

a) $\lim\limits_{x\to 3}\frac{5x+11}{\sqrt{x+1}} = \frac{15+11}{2} = 13$

b) $\lim\limits_{x\to \frac{3}{2}}\frac{2x^2+x-6}{4x^2-4x-3} = \lim\limits_{x\to \frac{3}{2}}\frac{\cancel{(2x-3)}(x+2)}{(2x+1)\cancel{(2x-3)}} =$

$= \lim\limits_{x\to \frac{3}{2}}\frac{x+2}{2x+1} = \frac{\frac{3}{2}+2}{3+1} = \frac{\frac{7}{2}}{4} = \frac{7}{8}$

c) $\lim\limits_{x\to 0^+}\frac{1}{\sqrt{x}} = \infty$

$x>0$

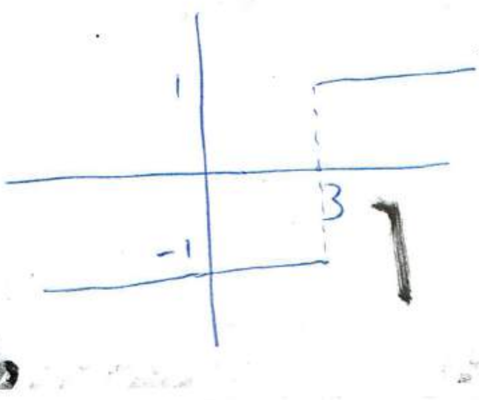
d) $\lim\limits_{x\to 5^-}\sqrt{5-x} = 0$

$5-x>0$
$x<5$

e) $\lim\limits_{x\to 3^+}\frac{3-x}{|3-x|} = 1$

f) $\lim\limits_{x\to 3^-}\frac{3-x}{|3-x|} = -1$

g) $\lim\limits_{x\to 3}\frac{3-x}{|3-x|} = DNE$



h) $\lim\limits_{x\to \infty}\frac{2}{x^3} = 0$

i) $\lim\limits_{x\to \infty}\frac{2}{x^3}+\frac{3}{x}+5 = \lim\limits_{x\to\infty}\frac{2}{x^3} + \lim\limits_{x\to\infty}\frac{3}{x} + \lim\limits_{x\to\infty} 5 = 0+0+5 = 5$

j) $\lim\limits_{x\to \infty}\frac{4x^4+x^3}{8x^4-3} = \frac{1}{2}$

k) $\lim\limits_{x\to \infty}\frac{-3x^2-7x}{x-1} = -\infty$

l) $\lim\limits_{x\to 0}\frac{4-\sqrt{16+x}}{x} = \lim\limits_{x\to 0}\frac{(4-\sqrt{16+x})(4+\sqrt{16+x})}{x(4+\sqrt{16+x})}$

$= \lim\limits_{x\to 0}\frac{16-(16+x)}{x(4+\sqrt{16+x})} = \lim\limits_{x\to 0}\frac{-\cancel{x}}{\cancel{x}(4+\sqrt{16+x})} = -$

m) $\lim\limits_{x\to 4^-}\frac{1}{x-4} = -\infty$

n) $\lim\limits_{x\to 4}\frac{1}{x-4} = DNE$

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