Congruent Triangles Investigation Date: ____________

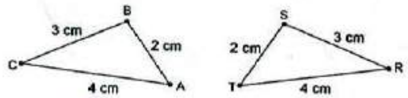
1. What does it mean to say two triangles are congruent?

   All pairs of corr. ∡s are ≅.
   All pairs of corr. sides are ≅.

2. List the ways to justify that triangles are similar.

   AA~, SAS~, SSS~

3. Examine the triangles with all side lengths labeled.



SSS

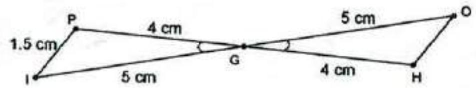
   a. How do we justify that the two triangles are similar? SSS~
   b. Complete the similarity statement: $\Delta ABC \sim \Delta$ TSR.
   c. What is the scale factor? 1 : 1
   d. What do we know about the corresponding angles of similar triangles?

      They are ≅.

   e. What does this tell us about the pair of triangles?

      They are ≅.

4. Examine the triangles with two sides lengths and an included angle labeled.

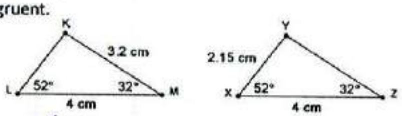


SAS

   a. How do we justify that the two triangles are similar? SAS~
   b. Complete the similarity statement: $\Delta PIG \sim \Delta$ HOG
   c. What is the scale factor? 1 : 1
   d. Since the triangles are similar, what do we know about $\angle P$ and its corresponding angle? ≅
      What do we know about $\angle I$ and its corresponding angle? ≅
   e. Use the scale factor you gave in part c to determine the length of $\overline{OH}$.
      1.5 cm
   f. What does this tell us about the pair of triangles?

      They are ≅.

5. Examine the triangles with two angle pairs marked congruent.



   a. How do we justify that the two triangles are similar? AA ~
   b. Complete the similarity statement: $\Delta KLM \sim \Delta$ YXZ.
   c. What is the scale factor? 1 : 1
   d. Since the triangles are similar, what do we know about $\angle K$ and $\angle Y$? ≅

   ASA

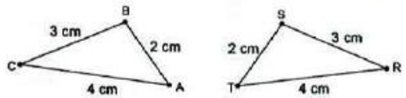
   e. Use the scale factor you gave in part c to determine the lengths of $\overline{KL}$ and $\overline{YZ}$.

      KL = 2.15cm YZ = 3.2cm

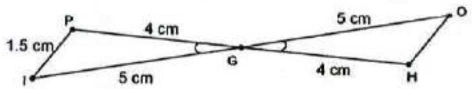
   f. What does this tell us about the pair of triangles?

      They are ≅

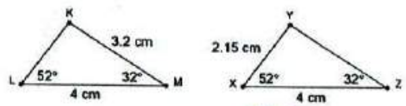
6. Think back to the three situations we examined.
   - In #3, we were given 3 pairs of sides of one triangle are congruent to 3 pairs of sides of another triangle. We were able to conclude that the triangles are ≅.



   - In #4, we were given 2 pairs of sides and an included angle of one triangle are congruent to 2 pairs of sides and an included angle of another triangle. We were able to conclude that the triangles are ≅.



   - In #5, we were given 2 pairs of angles and an included side of one triangle are congruent to 2 pairs of angles and an included side of another triangle. We were able to conclude that the triangles are ≅.

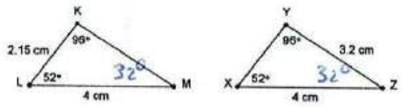


This investigation illustrates 3 of the shortcuts to prove that triangles are ≅.
Those shortcuts are SSS, SAS and ASA.

7. What if different parts of the 2 triangles are given congruent? Suppose rather than being given 2 pairs of angles and an included side of one triangle congruent to the corresponding parts of the 2nd triangle as in #5, we were instead given 2 pairs of angles and a non-included side of triangle congruent to 2 pairs of angles and a non-included side of another triangle. Could the triangles still be proven congruent? Examine the diagram below where this is the given information.

a. What is $m\angle M$? 32°

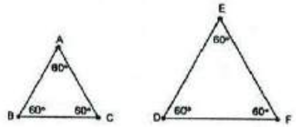
b. What is $m\angle Z$? 32°



c. Do you now have enough information to prove the triangles congruent by one of the shortcut methods from earlier in this investigation? Yes
If so, which one? ASA

d. In this diagram, you started with 2 pairs of angles and a non-included side of each triangle being congruent to their corresponding parts in the other triangle. The name for this shortcut is AAS.

8. Since we have now seen that SSS, SAS, ASA, and AAS are shortcuts to proving that triangles are congruent (without needing all 6 pairs of corresponding parts congruent to prove the triangles congruent by definition), one might wonder if there are additional shortcuts that will work. Students often wonder if AAA will work since SSS does or if SSA will work since AAS does.



a. Let's start by thinking about whether AAA would work. Using the diagram at the right, you can see that 3 angles of ΔABC are congruent to 3 angles of ΔDEF.

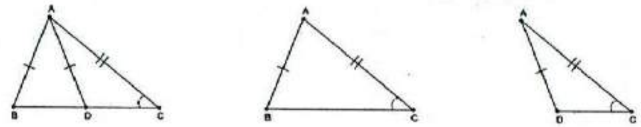
These triangles are drawn to scale. (They really are equilateral triangles.)

AAA no!

Are their corresponding sides congruent? no
Are the triangles congruent? no
Is AAA an appropriate shortcut to prove triangles congruent? no

(If it were, all equilateral triangles would be congruent to one another, and we know that isn't the case!)

b. Now let's examine whether SSA is sufficient to guarantee congruent triangles. The 3 diagrams below show ΔABC and ΔADC drawn to scale. The first diagram is the original figure and shows that the triangles are overlapping. In the 2nd and 3rd diagrams, they are separated out so it is easier to see the parts that are marked congruent on each separate triangle.



Notice that two sides and a non-included angle of one of the triangles (ΔABC) are congruent to two sides and a non-included angle of the other triangle (ΔADC). That means that the given congruent parts are a SSA situation.

i. What do you know about ΔABC and ΔADC from the first diagram?

$\overline{AB} \cong \overline{AD}$ $\angle C \cong \angle C$ SSA no!
$\overline{AC} \cong \overline{AC}$

ii. Based on your answer to part i, is it possible for ΔABC and ΔADC to be congruent? ________

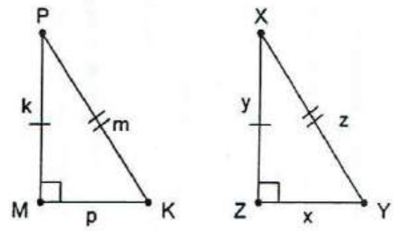
iii. Therefore, does SSA guarantee congruent triangles? no

iv. If you need more convincing, use patty paper to trace ΔABC and ΔADC from their separate diagrams above and compare to those triangles in the 1st overlapping diagram. This should make it clear that we should never use SSA to try to prove congruent triangles. This does not mean that when given these parts, triangle cannot be congruent. Certainly if we had congruent triangles, we could correctly mark 2 sides and a non-included angle of one triangle congruent to the corresponding parts of the other triangle. It does mean, however, that if we only have those parts available, we cannot conclude definitively that the triangles are congruent. They might be, but they might not be. You will study the various possibilities when these corresponding parts are known to be congruent in more depth in a later course.

9. You have now examined 6 different situations in this investigation. Before investigating one last situation, think (or look) back. Instead of needing all 6 pairs of corresponding parts congruent to prove triangles congruent by definition, what are 4 valid shortcuts for proving triangles congruent?

SSS, SAS, ASA, and AAS.

10. In #8, we established that SSA does not guarantee congruent triangles. Look at when 2 right triangles are used in a similar situation where the two pairs of congruent sides are a pair of legs and the hypotenuses and the congruent angle pair is the right angles as shown in the diagram.



a. What theorem do you know that relates the 3 sides of a right triangle?

Pythagorean theorem

b. Use that theorem to write an equation relating the sides of ΔPKM. Then solve that equation for the length of side *p*.

$k^2 + p^2 = m^2$

$p^2 = m^2 - k^2$

$\sqrt{p^2} = \sqrt{m^2 - k^2}$

$p = \sqrt{m^2 - k^2}$

c. Use that same theorem to write an equation relating the sides of ΔXYZ. Do not solve it!

$y^2 + x^2 = z^2$

d. Based on the given information in the diagram, what can you substitute in place of *y* in the equation above? k What can you substitute in place of *z*? m Rewrite your equation from part c above using these substitutions.

$k^2 + x^2 = m^2$

e. Now solve the equation in part d above for the length of side *x*.

$x^2 = m^2 - k^2$

$\sqrt{x^2} = \sqrt{m^2 - k^2}$

$x = \sqrt{m^2 - k^2}$

f. How do the lengths of side *p* and side *x* relate to one another? They are $\cong$

Use this idea to appropriately mark the sides in the diagram above.

g. Based on the markings in the diagram above, what shortcut from earlier in this investigation can you use to prove that ΔPKM and ΔXYZ are congruent?

SSS

h. Since we can use the Pythagorean Theorem to find the 3rd side of a right triangle when 2 sides are known, we were able to determine that the 2 triangles above are congruent. This illustrates why a fifth shortcut to prove triangles congruent exists. It is called HL, which stands for Hypotenuse-Leg Theorem. It states that if the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent. Remember that only right triangles can be proven using this theorem!

11. List the 5 shortcuts you have explored that can be used to prove triangles congruent:

SSS, SAS, ASA, AAS, and HL

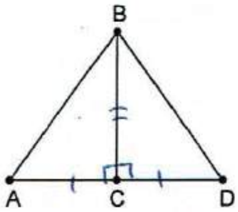
10

Ways to Prove Triangles Congruent Date: ____________

Shortcuts to Prove Triangles are Congruent: SSS, SAS, ASA, AAS, HL

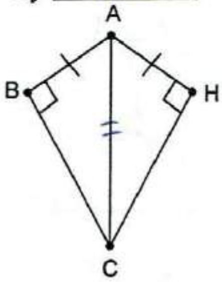
Examples:

1. $\Delta ABC \cong \Delta$ DBC
by SAS

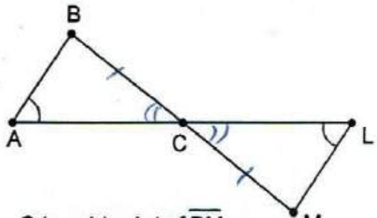


$\overline{BC}$ is $\perp$ bis of $\overline{AD}$

4. $\Delta ABC \cong \Delta$ AHC
by HL

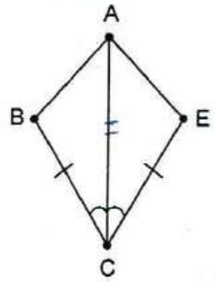


7. $\Delta ABC \cong \Delta$ LMC
by AAS

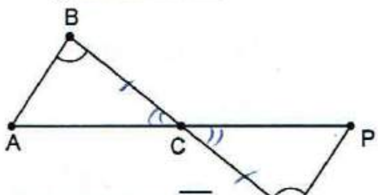


C is midpoint of $\overline{BM}$

2. $\Delta ABC \cong \Delta$ AEC
by SAS

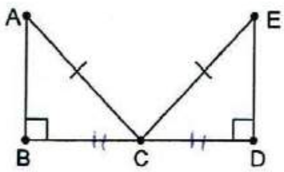


5. $\Delta ABC \cong \Delta$ PQC
by ASA



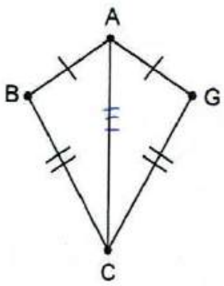
C is midpoint of $\overline{BQ}$

8. $\Delta ABC \cong \Delta$ EDC
by HL



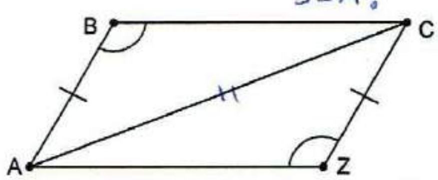
C is midpoint of $\overline{BD}$

3. $\Delta ABC \cong \Delta$ AGC
by SSS

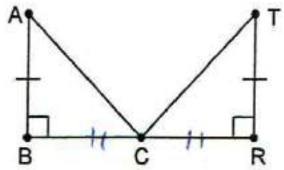


6. $\Delta ABC \cong \Delta$ none
by none

SSA!



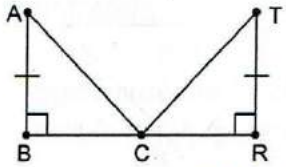
9. $\Delta ABC \cong \Delta$ TRC
by SAS



C is midpoint of $\overline{BR}$

Mark any additional congruence you can determine in the diagram. Then write any additional congruent parts that are needed to prove the triangles are congruent by the given method.

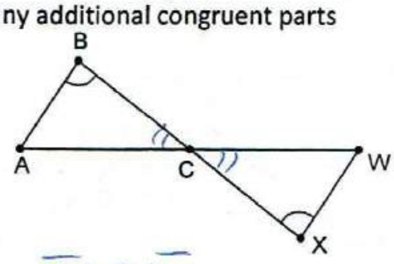
10.



a. SAS $\overline{BC} \cong \overline{RC}$

b. ASA $\angle A \cong \angle T$

11.



a. ASA $\overline{BC} \cong \overline{XC}$

b. AAS $\overline{AC} \cong \overline{WC}$ or $\overline{AB} \cong \overline{WX}$

12