

## Differentiation - Product Rule

**Differentiate each function with respect to  $x$ .**

1)  $y = -x^3(3x^4 - 2)$

$$\begin{aligned}\frac{dy}{dx} &= -x^3 \cdot 12x^3 + (3x^4 - 2) \cdot -3x^2 \\ &= -21x^6 + 6x^2\end{aligned}$$

2)  $f(x) = x^2(-3x^2 - 2)$

$$\begin{aligned}f'(x) &= x^2 \cdot -6x + (-3x^2 - 2) \cdot 2x \\ &= -12x^3 - 4x\end{aligned}$$

3)  $y = (-2x^4 - 3)(-2x^2 + 1)$

$$\begin{aligned}\frac{dy}{dx} &= (-2x^4 - 3) \cdot -4x + (-2x^2 + 1) \cdot -8x^3 \\ &= 24x^5 - 8x^3 + 12x\end{aligned}$$

4)  $f(x) = (2x^4 - 3)(x^2 + 1)$

$$\begin{aligned}f'(x) &= (2x^4 - 3) \cdot 2x + (x^2 + 1) \cdot 8x^3 \\ &= 12x^5 + 8x^3 - 6x\end{aligned}$$

5)  $f(x) = (5x^5 + 5)(-2x^5 - 3)$

$$\begin{aligned}f'(x) &= (5x^5 + 5) \cdot -10x^4 + (-2x^5 - 3) \cdot 25x^4 \\ &= -100x^9 - 125x^4\end{aligned}$$

6)  $f(x) = (-3 + x^{-3})(-4x^3 + 3)$

$$\begin{aligned}f'(x) &= (-3 + x^{-3}) \cdot -12x^2 + (-4x^3 + 3) \cdot -3x^{-4} \\ &= 36x^2 - \frac{9}{x^4}\end{aligned}$$

7)  $y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$

$$\begin{aligned}\frac{dy}{dx} &= (-2x^4 + 5x^2 + 4) \cdot -6x + (-3x^2 + 2)(-8x^3 + 10x) \\ &= 36x^5 - 76x^3 - 4x\end{aligned}$$

8)  $y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$

$$\begin{aligned}\frac{dy}{dx} &= (x^4 + 3)(-20x^4 + 20x^3) + (-4x^5 + 5x^4 + 5) \cdot 4x^3 \\ &= -36x^8 + 40x^7 - 60x^4 + 80x^3\end{aligned}$$

$$9) y = (5x^4 - 3x^2 - 1)(-5x^2 + 3)$$

$$\begin{aligned} \frac{dy}{dx} &= (5x^4 - 3x^2 - 1) \cdot -10x + (-5x^2 + 3)(20x^3 - 6x) \\ &= -150x^5 + 120x^3 - 8x \end{aligned}$$

$$10) f(x) = (-10x^2 - 7\sqrt[5]{x^2} + 9)(2x^3 + 4)$$

$$\begin{aligned} f'(x) &= \left(-10x^2 - 7x^{\frac{2}{5}} + 9\right) \cdot 6x^2 + (2x^3 + 4) \left(-20x - \frac{14}{5}x^{-\frac{3}{5}}\right) \\ &= -100x^4 - \frac{238x^{\frac{12}{5}}}{5} + 54x^2 - 80x - \frac{56}{5x^{\frac{3}{5}}} \end{aligned}$$

$$11) y = (5 + 3x^{-2})(4x^5 + 6x^3 + 10)$$

$$\begin{aligned} \frac{dy}{dx} &= (5 + 3x^{-2})(20x^4 + 18x^2) + (4x^5 + 6x^3 + 10) \cdot -6x^{-3} \\ &= 100x^4 + 126x^2 + 18 - \frac{60}{x^3} \end{aligned}$$

$$12) y = (-6x^4 + 2 + 6x^{-4})(6x^4 + 7)$$

$$\begin{aligned} \frac{dy}{dx} &= (-6x^4 + 2 + 6x^{-4}) \cdot 24x^3 + (6x^4 + 7)(-24x^3 - 24x^{-5}) \\ &= -288x^7 - 120x^3 - \frac{168}{x^5} \end{aligned}$$

$$13) f(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right)(x^2 + 10)$$

$$\begin{aligned} f'(x) &= \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right) \cdot 2x + (x^2 + 10) \left(-28x^3 + 4x^{-\frac{3}{5}}\right) \\ &= -42x^5 - 280x^3 + 24x^{\frac{7}{5}} + 16x + \frac{40}{x^{\frac{3}{5}}} \end{aligned}$$

### Critical thinking question:

14) A classmate claims that  $(f \cdot g)' = f' \cdot g'$  for any functions  $f$  and  $g$ . Show an example that proves your classmate wrong.

Many answers. Ex:  $f = 2x$ ,  $g = 4$ ,  $8 \neq 0$