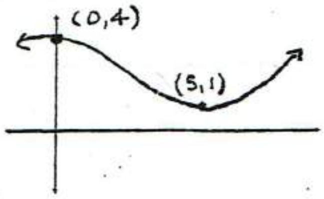
ICM

Intro to Limits of Functions

1. Graph is continuous & smooth.



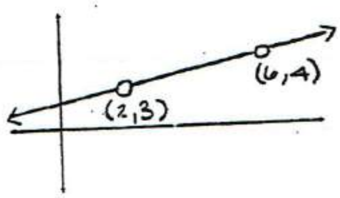
a. $\lim_{x\to 0} f(x) =$ 4

b. $\lim_{x\to 5} f(x) =$ 1

c. $f(0) =$ 4

d. $f(4) =$ 1

2. Graph has a single point hole.



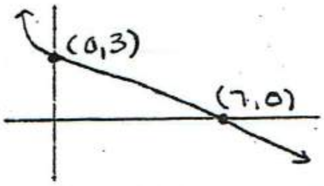
a. $\lim_{x\to 2} f(x) =$ 3

b. $\lim_{x\to 6} f(x) =$ 4

c. $f(2) =$ undefined

d. $f(6) =$ undefined

3. Graph is continuous & smooth.



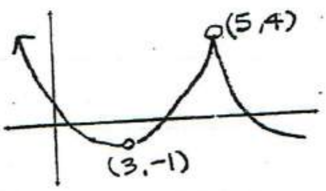
a. $\lim_{x\to 0} f(x) =$ 3

b. $\lim_{x\to 7} f(x) =$ 0

c. $f(0) =$ 3

d. $f(7) =$ 0

4. Graph has a single point hole.



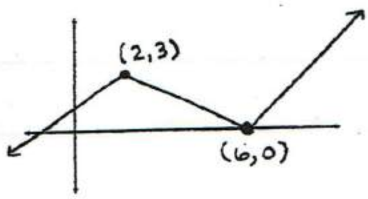
a. $\lim_{x\to 3} f(x) =$ -1

b. $\lim_{x\to 5} f(x) =$ 4

c. $f(3) =$ undefined

d. $f(5) =$ undefined

5. Graph has a sharp bend.



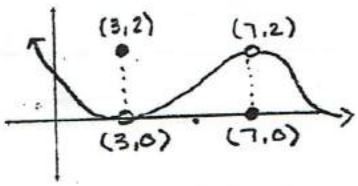
a. $\lim_{x\to 2} f(x) =$ 3

b. $\lim_{x\to 6} f(x) =$ 0

c. $f(2) =$ 3

d. $f(6) =$ 0

6. Graph has a single point hole.



a. $\lim_{x\to 3} f(x) =$ 0

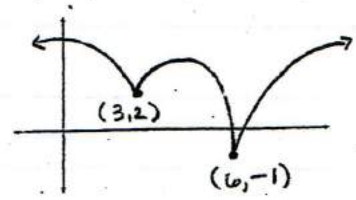
b. $\lim_{x\to 7} f(x) =$ 2

c. $f(3) =$ 2

d. $f(7) =$ 0

Adapted from Dr. Love, NCSU
MAT 191

7. Graph has sharp bend.



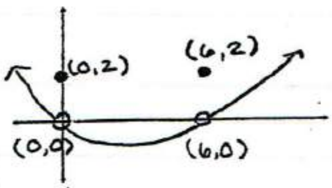
a. $\lim_{x\to 3} f(x) = 2$

b. $\lim_{x\to 6} f(x) = -1$

c. $f(3) = 2$

d. $f(6) = -1$

8. Graph has single point hole.



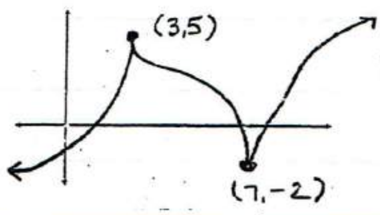
a. $\lim_{x\to 0} f(x) = 0$

b. $\lim_{x\to 6} f(x) = 0$

c. $f(0) = 2$

d. $f(6) = 2$

9. Graph has sharp bend.



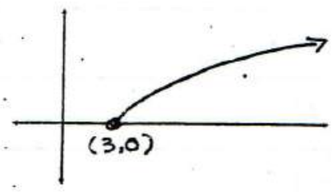
a. $\lim_{x\to 3} f(x) = 5$

b. $\lim_{x\to 7} f(x) = -2$

c. $f(3) = 5$

d. $f(7) = -2$

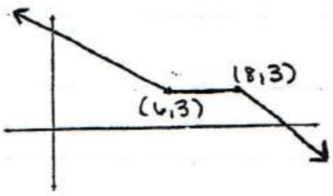
10. Graph has endpoints.



a. $\lim_{x\to 3} f(x) =$ DNE

b. $f(3) = 0$

11. Graph has sharp bend.



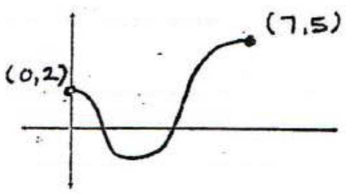
a. $\lim_{x\to 6} f(x) = 3$

b. $\lim_{x\to 8} f(x) = 3$

c. $f(6) = 3$

d. $f(8) = 3$

12. Graph has endpoints.



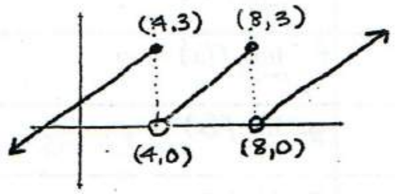
a. $\lim_{x\to 0^+} f(x) = 2$

b. $\lim_{x\to 0} f(x) =$ DNE

c. $\lim_{x\to 7^-} f(x) = 5$

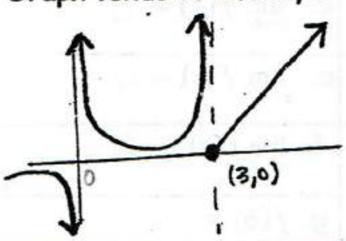
d. $\lim_{x\to 7} f(x) =$ DNE

13. Graph has gap-jump.



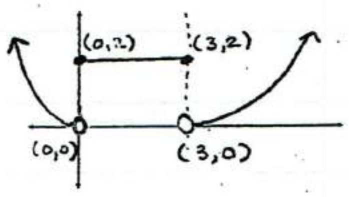
a. $\lim_{x \to 4^-} f(x) =$ 3	e. $\lim_{x \to 8^-} f(x) =$ 3
b. $\lim_{x \to 4^+} f(x) =$ 0	f. $\lim_{x \to 8^+} f(x) =$ 0
c. $\lim_{x \to 4} f(x) =$ DNE	g. $\lim_{x \to 8} f(x) =$ DNE
d. $f(4) =$ 3	h. $f(8) =$ 3

14. Graph tends to infinity.



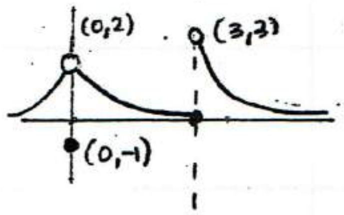
a. $\lim_{x \to 0^-} f(x) =$ $-\infty$	e. $\lim_{x \to 3^-} f(x) =$ ∞
b. $\lim_{x \to 0^+} f(x) =$ ∞	f. $\lim_{x \to 3^+} f(x) =$ 0
c. $\lim_{x \to 0} f(x) =$ DNE	g. $\lim_{x \to 3} f(x) =$ DNE
d. $f(0) =$ undefined	h. $f(3) =$ 0

15. Graph has a gap-jump.



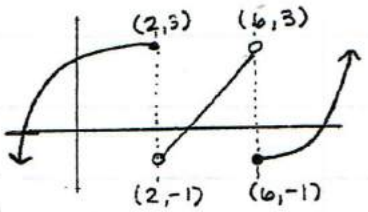
a. $\lim_{x \to 0^-} f(x) =$ 0	e. $\lim_{x \to 3^-} f(x) =$ 2
b. $\lim_{x \to 0^+} f(x) =$ 2	f. $\lim_{x \to 3^+} f(x) =$ 0
c. $\lim_{x \to 0} f(x) =$ DNE	g. $\lim_{x \to 3} f(x) =$ DNE
d. $f(0) =$ 2	h. $f(3) =$ 2

16.



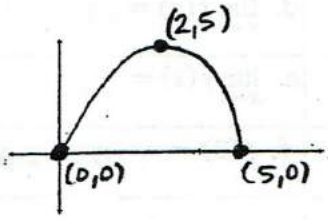
a. $\lim_{x \to 0^-} f(x) =$ 2	e. $\lim_{x \to 3^-} f(x) =$ 0
b. $\lim_{x \to 0^+} f(x) =$ 2	f. $\lim_{x \to 3^+} f(x) =$ 3
c. $\lim_{x \to 0} f(x) =$ 2	g. $\lim_{x \to 3} f(x) =$ DNE
d. $f(0) =$ -1	h. $f(3) =$ 0

17. Graph has a gap-jump.



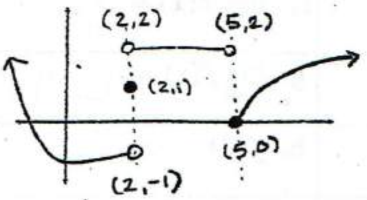
a. $\lim_{x \to 2^-} f(x) =$ 3	e. $\lim_{x \to 6^-} f(x) =$ 3
b. $\lim_{x \to 2^+} f(x) =$ -1	f. $\lim_{x \to 6^+} f(x) =$ -1
c. $\lim_{x \to 2} f(x) =$ DNE	g. $\lim_{x \to 6} f(x) =$ DNE
d. $f(2) =$ 3	h. $f(6) =$ -1

18.



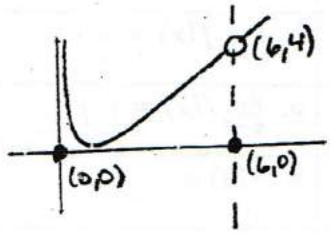
a. $\lim_{x \to 2^-} f(x) =$ 5	e. $\lim_{x \to 5^-} f(x) =$ 0
b. $\lim_{x \to 2^+} f(x) =$ 5	f. $f(5) =$ 0
c. $\lim_{x \to 2} f(x) =$ 5	g. $f(0) =$ 0
d. $f(2) =$ 5	

19. Graph has gap-jump.



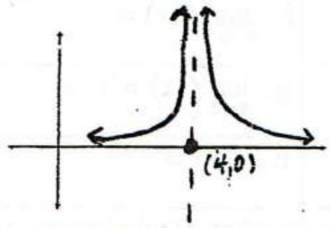
a. $\lim_{x\to 2^-} f(x) =$ -1	e. $\lim_{x\to 5^-} f(x) =$ 2
b. $\lim_{x\to 2^+} f(x) =$ 2	f. $\lim_{x\to 5^+} f(x) =$ 0
c. $\lim_{x\to 2} f(x) =$ DNE	g. $\lim_{x\to 5} f(x) =$ DNE
d. $f(2) =$ 1	h. $f(5) =$ 0

20.



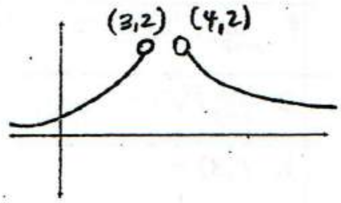
a. $\lim_{x\to 0^+} f(x) =$ ∞	d. $\lim_{x\to 6^-} f(x) =$ 4
b. $\lim_{x\to 0} f(x) =$ DNE	e. $\lim_{x\to 6^+} f(x) =$ 4
c. $f(0) =$ 0	f. $\lim_{x\to 6} f(x) =$ 4
	g. $f(6) =$ 0

21. Graph tends to infinity.



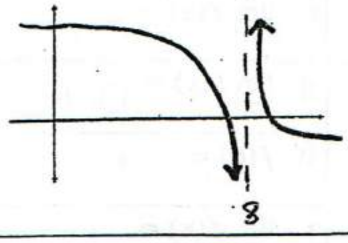
a. $\lim_{x\to 4^-} f(x) =$ ∞
b. $\lim_{x\to 4^+} f(x) =$ ∞
c. $\lim_{x\to 4} f(x) =$ ∞
d. $f(4) =$ 0

22.



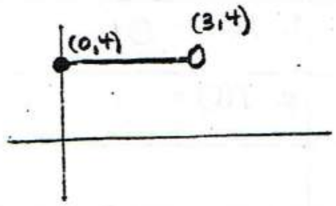
a. $\lim_{x\to 3^-} f(x) =$ 2
b. $\lim_{x\to 4^+} f(x) =$ 2
c. $f(3) =$ undefined
d. $f(4) =$ undefined

23. Graph tends to infinity.

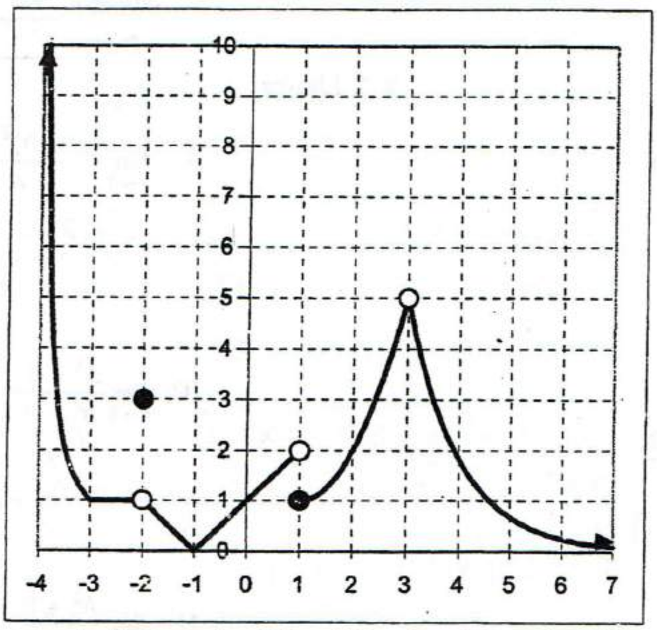


a. $\lim_{x\to 8^-} f(x) =$ $-\infty$
b. $\lim_{x\to 8^+} f(x) =$ ∞
c. $\lim_{x\to 8} f(x) =$ DNE
d. $f(8) =$ Undefined

24.



a. $\lim_{x\to 0^+} f(x) =$ 4	d. $\lim_{x\to 3^-} f(x) =$ 4
b. $\lim_{x\to 0} f(x) =$ DNE	e. $\lim_{x\to 3} f(x) =$ DNE
c. $f(0) =$ 4	f. $f(3) =$ undefined



Using the above graph, find each of the following (You should assume that y=0 is a horizontal asymptote and x = -4 is a vertical asymptote):

1) $f(-2) =$ 3

2) $\lim_{x\to -2^+} f(x) =$ 1

3) $\lim_{x\to -2} f(x) =$ 1

4) $\lim_{x\to -1^+} f(x) =$ 0

5) $\lim_{x\to -1^-} f(x) =$ 0

6) $\lim_{x\to -1} f(x) =$ 0

7) $\lim_{x\to 1^+} f(x) =$ 1

8) $\lim_{x\to 1^-} f(x) =$ 2

9) $\lim_{x\to 1} f(x) =$ DNE

10) $f(3) =$ undefined

11) $\lim_{x\to 3^+} f(x) =$ 5

12) $\lim_{x\to 3^-} f(x) =$ 5

13) $\lim_{x\to 3} f(x) =$ 5

14) $\lim_{x\to -4^+} f(x) =$ ∞

15) $\lim_{x\to \infty} f(x) =$ 0

16) $f(1) =$ 1

17) $\lim_{x\to 3} f(x) =$ 1

18) $f(-4) =$ undefined

For each of the following problems, find the requested limit.

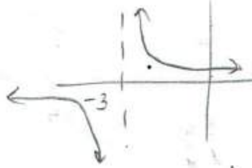
1) $\lim_{x\to 2} 7 =$ 7

2) $\lim_{x\to 5} \sqrt{x-2} =$ $\sqrt{5-2} = \sqrt{3}$

3) $\lim_{x\to -5} \dfrac{x^2-25}{x+5} =$ $\cancel{\dfrac{25-25}{-5+5} = \dfrac{0}{0}}$! $\lim_{x\to -5} \dfrac{(x-5)\cancel{(x+5)}}{\cancel{(x+5)}}$ $\lim_{x\to -5} (x-5)$

$-5-5 = -10$

4) $\lim_{x\to -3} \dfrac{x}{x+3} =$ $\cancel{\dfrac{-3}{-3+3} = \dfrac{-3}{0}}$!



DNE

Day 1

2.1 Limits

Find the limits, if they exist.

1. $\lim_{x\to 2} \frac{x^2-4}{x-2}$

$\lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$2+2 = 4$

2. $\lim_{x\to 3} \frac{2x^3-6x^2+x-3}{x-3}$

$\lim_{x\to 3} \frac{2x^2(x-3)+1(x-3)}{x-3}$

$\lim_{x\to 3} \frac{\cancel{(x-3)}(2x^2+1)}{\cancel{(x-3)}} = 2(3)^2+1 = 19$

3. $\lim_{x\to 1} \frac{x^2-x}{2x^2+5x-7}$

$\lim_{x\to 1} \frac{x\cancel{(x-1)}}{(2x+7)\cancel{(x-1)}} = \frac{1}{2(1)+7} = \frac{1}{9}$

4. $\lim_{r\to -3} \frac{r^2+2r-3}{r^2+7r+12}$

$\lim_{r\to -3} \frac{\cancel{(r+3)}(r-1)}{\cancel{(r+3)}(r+4)} = \frac{-3-1}{-3+4} = \frac{-4}{1} = -4$

5. $\lim_{x\to 5} \frac{3x^2-13x-10}{2x^2-7x-15}$

$\lim_{x\to 5} \frac{(3x+2)\cancel{(x-5)}}{(2x+3)\cancel{(x-5)}} = \frac{3(5)+2}{2(5)+3} = \frac{17}{13}$

6. $\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25}$

$\lim_{x\to 25} \frac{\cancel{\sqrt{x}-5}}{\cancel{(\sqrt{x}-5)}(\sqrt{x}+5)} = \frac{1}{\sqrt{25}+5} = \frac{1}{10}$

7. $\lim_{k\to 4} \frac{k^2-16}{\sqrt{k}-2}$

$\lim_{k\to 4} \frac{(k-4)(k+4)}{\sqrt{k}-2} = \lim_{k\to 4} \frac{\cancel{(\sqrt{k}-2)}(\sqrt{k}+2)(k+4)}{\cancel{(\sqrt{k}-2)}}$

$= (\sqrt{4}+2)(4+4) = 4(8) = 32$

8. $\lim_{h\to 0} \frac{(x+h)^3-x^3}{h}$

$\lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$

$= 3x^2+0+0$

$= 3x^2$

9. $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$

$\lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h}{\cancel{h}}$

$= 2x$

10. $\lim_{h\to 2} \frac{h^3-8}{h^2-4}$

$\lim_{h\to 2} \frac{\cancel{(h-2)}(h^2+2h+4)}{\cancel{(h-2)}(h+2)} = \frac{4+4+4}{4}$

$= 3$

11. $\lim_{h\to 0} \frac{h^3+8}{h+2}$

$\lim_{h\to 0} \frac{\cancel{(h+2)}(h^2-2h+4)}{\cancel{h+2}} = 4$

12. $\lim_{z\to 10} \frac{1}{z-10}$ DNE

13. $\lim_{x\to -3/2} \frac{2x+3}{4x^2+12x+9}$

$\lim_{x\to -3/2} \frac{\cancel{2x+3}}{\cancel{(2x+3)}(2x+3)} = \text{DNE}$

14. $\lim_{s\to -1} \frac{1}{s^2+2s+1}$

$\lim_{s\to -1} \frac{1}{(s+1)(s+1)} = \infty$

15. $\lim_{x\to 0} \frac{1}{x^2} = \infty$

16. $\lim_{t\to 1} \frac{\frac{1}{t}-1}{t-1}$ $\frac{t}{t}$

$\lim_{t\to 1} \frac{1-t}{t(t-1)}$

$\lim_{t\to 1} \frac{-\cancel{(t-1)}}{t\cancel{(t-1)}} = -1$

$\frac{|x|}{x}$ $\sqrt{0}$ $\frac{1}{x}$ $\frac{1}{x^2}$

ICM
Calculus

Name: ______________________
Date: ______________ Pd: ______

2.4 Limits

Find the limit, if it exists.

1. $\lim\limits_{x\to 0^+}\left(4+\sqrt{x}\right)$ $= 4$

   $4+\sqrt{0}$

2. $\lim\limits_{x\to 0^+}\left(-\sqrt{x}+3\right)$ $= 3$

   $-\sqrt{0}+3$

3. $\lim\limits_{x\to -6^+}\left(\sqrt{x+6}+x\right)$ $= -6$

   $\sqrt{0}+-6$

4. $\lim\limits_{x\to \frac{5}{2}^-}\left(\sqrt{5-2x}-x^2\right)$ $= -\frac{25}{4}$

   $\sqrt{0}-\frac{25}{4}$

5. $\lim\limits_{x\to 5^+}\left(\sqrt{x^2-25}+3\right)$ $= 3$

   $\sqrt{0}+3$

6. $\lim\limits_{x\to 3^-} x\sqrt{9-x^2}$ $= 0$

   $3\sqrt{0}$

7. $\lim\limits_{x\to 3^-}\frac{|x-3|}{x-3}$ $= -1$

8. $\lim\limits_{x\to -10^+}\frac{x+10}{|x+10|}$ $= 1$

9. $\lim\limits_{x\to -5^+}\frac{|x+5|}{x+5}$ $= 1$

10. $\lim\limits_{x\to 2^-}\frac{x-2}{|x-2|}$ $= -1$

11. $\lim\limits_{x\to 0^+}\frac{1}{x^2}$ $= \infty$

12. $\lim\limits_{x\to 4^+}\frac{2}{x+4}$ $= \frac{2}{4+4} = \frac{1}{4}$

13. $\lim\limits_{x\to 7^-}\frac{-3}{(x-7)^2}$ $= -\infty$

14. $\lim\limits_{x\to \pi^-}\frac{|\pi-x|}{x-\pi}$ $= 1$

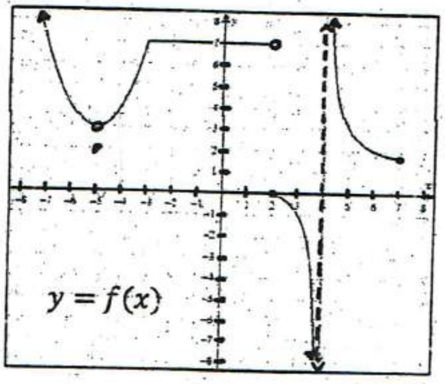
15. $\lim\limits_{x\to 0^+}\frac{1}{x}$ $= \infty$

16. $\lim\limits_{x\to 8^-}\frac{1}{x-8}$ $= -\infty$

Show all of your work! No work will receive little or no credit.

Calculators should NOT be used on this assignment.

In exercises 1 – 5, use the graph of $y = f(x)$ below, to evaluate the limits.



1. $\lim_{x\to 0} f(x) =$ 7
2. $\lim_{x\to -5} f(x) =$ 3
3. $\lim_{x\to 4} f(x) =$ DNE
4. $\lim_{x\to 2^+} f(x) =$ 0
5. $\lim_{x\to -4} f(x) =$ 4

Evaluate each of the following limits <u>ANALYTICALLY</u>.

6. $\lim_{x\to 3}(2x^2 - 5x + 4)$

$2(9) - 15 + 4 = 7$

7. $\lim_{x\to 36}\frac{\sqrt{x}-6}{x-36}$

$\lim_{x\to 36}\frac{\cancel{\sqrt{x}-6}}{(\cancel{\sqrt{x}-6})(\sqrt{x}+6)} = \frac{1}{6+6}$

$= \frac{1}{12}$

8. $\lim_{t\to -2}\frac{t^3+8}{t+2}$

$\lim_{t\to -2}\frac{\cancel{(t+2)}(t^2-2t+4)}{\cancel{t+2}}$

$= 4+4+4 = 12$

9. $\lim_{x\to 3}\frac{1}{(x-3)^2} = -\infty$

10. $\lim_{x\to -2^+}\sqrt{x+2} = 0$

-2

11. $\lim_{x\to 0}\frac{(2+x)^3 - 8}{x}$

$\lim_{x\to 0}\frac{8+12x+6x^2+x^3-8}{x}$

$\lim_{x\to 0}\frac{\cancel{x}(12+6x+x^3)}{\cancel{x}} = 12$

1 3 3 1

$1(2)^3 + 3(2)^2(x) + 3(2)(x)^2 + x^3$

Jean Adams