

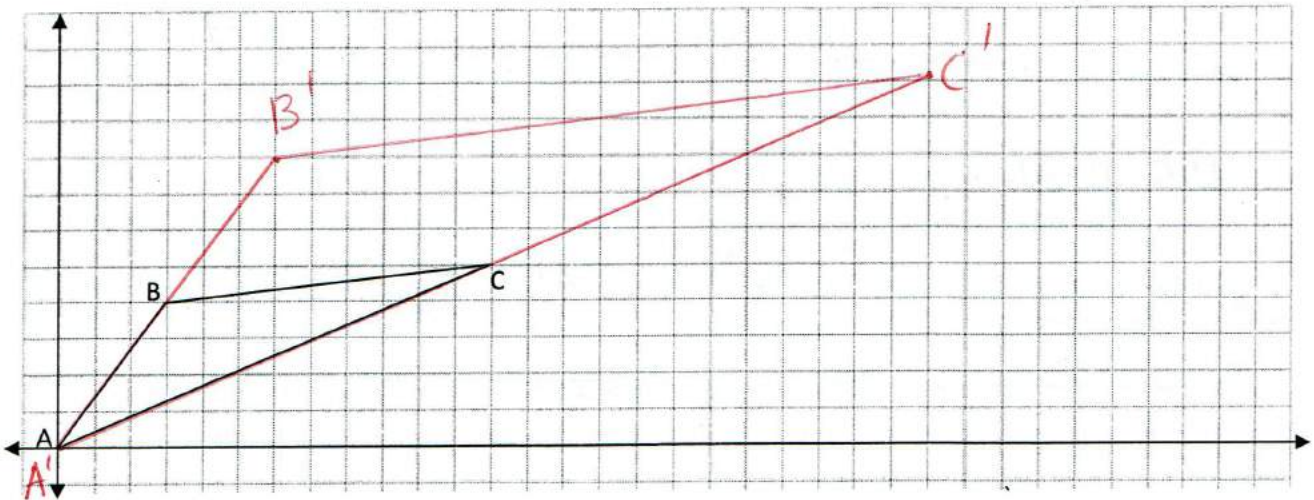
# UNIT 2 Parallel Lines & Similarity

Properties of Dilations Investigation

**STANDARD**

Date: \_\_\_\_\_

Dilate  $\triangle ABC$  about the origin with a magnitude of 2. Graph the new triangle; label the vertices  $A'$ ,  $B'$ , &  $C'$ .



Complete the following using your dilation.

- Using patty paper to compare the angles of  $\triangle ABC$  and  $\triangle A'B'C'$ . What do you notice?

Corresponding  $\angle$  pairs are  $\cong$ .

- Using the distance formula, calculate the lengths of  $AB$ ,  $A'B'$ ,  $AC$ ,  $A'C'$ ,  $BC$ , and  $B'C'$ . What do you notice? *Pyth. Thm.*

$$AB = 5$$

$$A'B' = 10$$

$$AC = 13$$

$$A'C' = 26$$

$$BC = \sqrt{82}$$

$$B'C' = \sqrt{328}$$

$$= 2\sqrt{82}$$

Lengths of the images are double the lengths of preimages.

- Dilations create **similar figures**. Based on your observations from 1 and 2, what can we say about similar figures?

1) Corr.  $\angle$ s are  $\cong$ .

2) Corr. sides are proportional.

- What do you notice about  $\overline{AB}$  and  $\overline{A'B'}$ ?  $\overline{AC}$  and  $\overline{A'C'}$ ? Note that  $A$  and  $A'$  lie on the origin. What conclusion can you make about the segments of an image when the corresponding segments of the preimage pass through the center of dilation?

If the preimage passes through the center of dilation, then the image is on the same line.

- Using the slope formula, calculate the slopes of  $\overline{BC}$  and  $\overline{B'C'}$ . What do you notice? What conclusion can you make about the segments of an image when the corresponding segments of the preimage do not pass through the center of dilation?

$$m_{\overline{BC}} = \frac{1}{9}$$

$$m_{\overline{B'C'}} = \frac{2}{18} = \frac{1}{9}$$

If the preimage does not pass through the center of dilation, then the image is parallel to it.

similar figures – figures whose corr.  $\angle$ s are  $\cong$  and whose corr. sides are proportional

If  $\triangle ABC \sim \triangle PQR$ , then:

$$\begin{aligned} \angle A &\cong \angle P \\ \angle B &\cong \angle Q \\ \angle C &\cong \angle R \end{aligned}$$

and

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

Examples:  $\triangle LMN \sim \triangle OPN$

1. Find x.

$$\frac{MN}{PN} = \frac{LN}{ON}$$

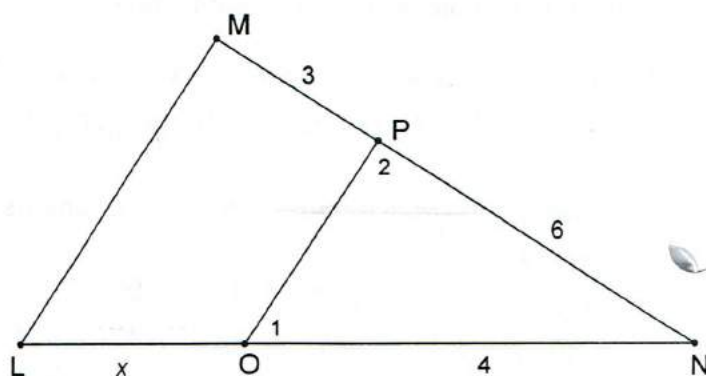
$$\frac{9}{6} = \frac{x+4}{4}$$

$$6(x+4) = 9 \cdot 4$$

$$6x + 24 = 36$$

$$6x = 12$$

$$\boxed{x = 2}$$



2. Find the scale factor of  $\triangle LMN : \triangle OPN$ .

$$\frac{MN}{PN} = \frac{9}{6} = \frac{3}{2}$$



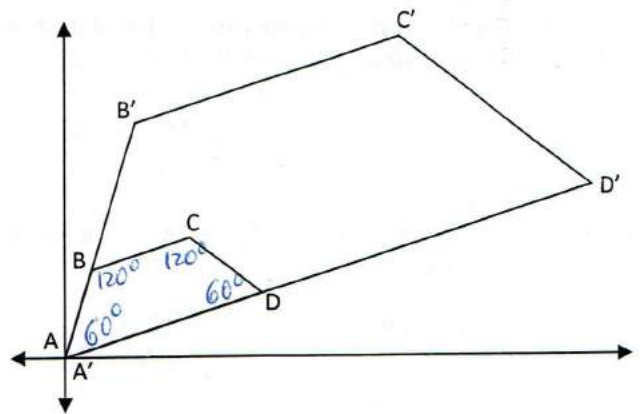
Dilations and Similar Figures

Date: \_\_\_\_\_

Dilations, Part 1:

Quadrilateral ABCD has been dilated about the origin by a magnitude of 3 to obtain Quadrilateral A'B'C'D'.

Use the information provided to answer each of the questions below.



1. If the measures of  $\angle ABC$  and  $\angle C$  are  $120^\circ$ , and the measures of  $\angle A$  and  $\angle ADC$  are  $60^\circ$ , what is the measure of  $\angle C'$ ? What is the measure of  $\angle A'$ ?   
 $\angle C' \cong \angle C$   
 $m\angle C' = m\angle C$

2. What is the ratio of  $CD : C'D'$ ? What is the ratio of  $A'B' : AB$ ?  
 $CD : C'D' = \frac{1}{3}$        $A'B' : AB = \frac{3}{1}$

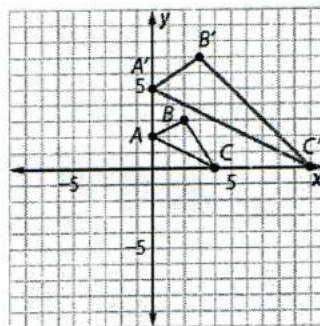
3. If  $AD$  is 8 in, what is the length of  $A'D'$ ?  
 $A'D' : AD = \frac{3}{1}$        $\frac{A'D'}{8} = \frac{3}{1}$        $A'D' = 8 \cdot 3 = 24$  in

4. If  $B'C'$  is 15 in, what is the length of  $BC$ ?  
 $B'C' : BC = \frac{3}{1}$        $\frac{15}{BC} = \frac{3}{1}$        $BC = \frac{15}{3} = 5$

5. Name the segments that are parallel to each other. How can you be sure?  
 $\overline{BC}$  and  $\overline{B'C'}$       if the preimage does not pass through the center of dilation, then the image is parallel to it.  
 $\overline{CD}$  and  $\overline{C'D'}$

6. If the slope of  $\overline{AB}$  is  $\frac{4}{3}$ , what is the slope of  $\overline{A'B'}$ ? How do you know this?  
 $\frac{4}{3}$       If the preimage passes through the center of dilation, then the image is on the same line.

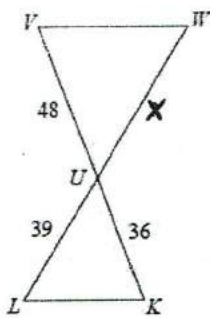
Part 2: Triangle ABC and  $\triangle A'B'C'$  are shown below. The scale on each axis is 1.



Since  $\triangle A'B'C'$  is bigger than  $\triangle ABC$ , Alex thinks that  $\triangle A'B'C'$  can be obtained by applying a size transformation centered at the origin to  $\triangle ABC$ . Do you agree or disagree with Alex? Explain your reasoning.

Similarity

1. Given that the triangles are similar, determine the scale factor and solve for the missing lengths.



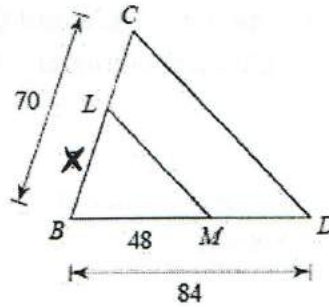
$$s.f. = \frac{48}{36} = \frac{4}{3}$$

$$\frac{4}{3} = \frac{X}{39}$$

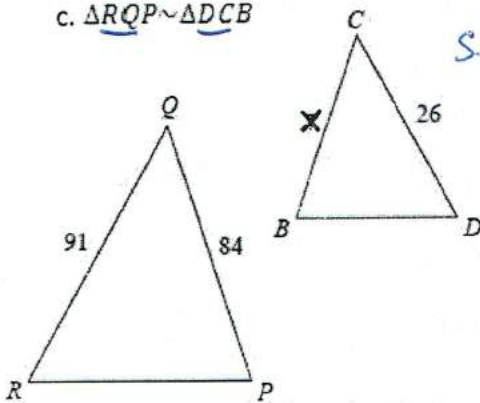
$$3X = 156$$

$$X = 52$$

- b.  $\Delta LBM \sim \Delta CBD$



- c.  $\Delta RQP \sim \Delta DCB$



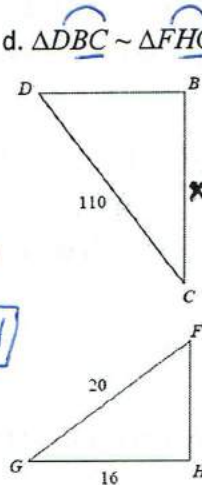
$$s.f. = \frac{91}{26} = \frac{7}{2}$$

$$\frac{84}{X} = \frac{7}{2}$$

$$7X = 168$$

$$X = 24$$

- d.  $\Delta DBC \sim \Delta FHG$



$$s.f. = \frac{110}{20} = \frac{11}{2}$$

$$\frac{X}{16} = \frac{11}{2}$$

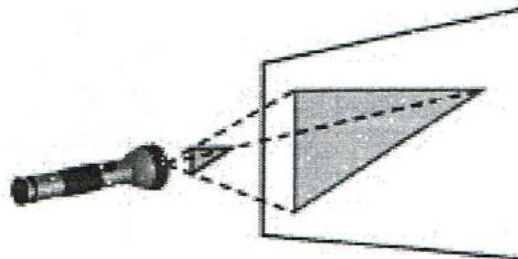
$$2X = 176$$

$$X = 88$$

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2. A flashlight is directed perpendicularly at a vertical wall 24 cm away. A cardboard triangle with sides of lengths 3, 4, and 5 cm is positioned directly between the light and the wall, parallel to the wall so that its projected shadow image is similar to it.

- Suppose the shadow of the 4-cm side is 10 cm. Find the lengths of the shadows of the other two sides.
- Suppose the shadow of the 5-cm side is 7.5 cm. Find the lengths of the other two sides of the shadow.



- How far from the light source should you place the cardboard triangle so that the 4-cm side has a 12-cm shadow?

Geometry Worksheet

Similar polygons

Name \_\_\_\_\_

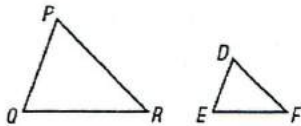
1. If polygons are similar then what do you know about the corresponding sides and the corresponding angles?

↓  
proportional

↓  
≅

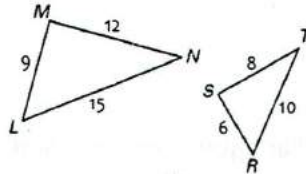
Given the similar figures, name all pairs of corresponding sides and angles. Look at the similarity statement to help.

2.  $\triangle PQR \sim \triangle DEF$



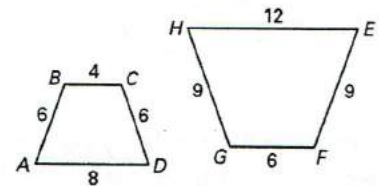
$$\begin{aligned} \overline{QP} &\rightarrow \overline{ED} & \angle Q &\cong \angle E \\ \overline{PR} &\rightarrow \overline{DF} & \angle P &\cong \angle D \\ \overline{RQ} &\rightarrow \overline{FE} & \angle R &\cong \angle F \end{aligned}$$

3.  $\triangle LMN \sim \triangle RST$



$$\begin{aligned} \overline{LM} &\rightarrow \overline{RS} & \angle L &\cong \angle R \\ \overline{MN} &\rightarrow \overline{ST} & \angle M &\cong \angle S \\ \overline{NL} &\rightarrow \overline{TR} & \angle N &\cong \angle T \end{aligned}$$

4.  $ABCD \sim HGFE$



$$\begin{aligned} \overline{AB} &\rightarrow \overline{HG} & \angle A &\cong \angle H \\ \overline{BC} &\rightarrow \overline{GF} & \angle B &\cong \angle G \\ \overline{CD} &\rightarrow \overline{FE} & \angle C &\cong \angle F \\ \overline{DA} &\rightarrow \overline{EH} & \angle D &\cong \angle E \end{aligned}$$

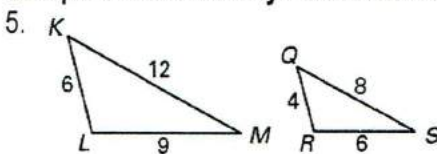
Use the similar polygons above to write the statement of proportionality for each:

$$\frac{QP}{ED} = \frac{PR}{DF} = \frac{RQ}{FE}$$

$$\frac{LM}{RS} = \frac{MN}{ST} = \frac{NL}{TR}$$

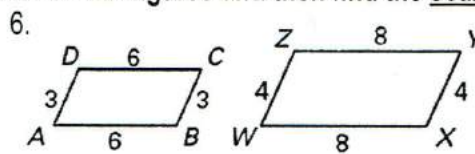
$$\frac{AB}{HG} = \frac{BC}{GF} = \frac{CD}{FE} = \frac{DA}{EH}$$

Complete the similarity statement for the similar figures and then find the scale factor. REDUCE fractions!



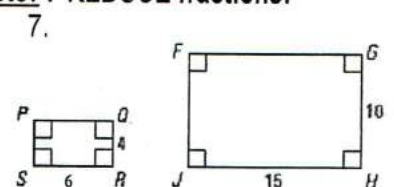
$$\triangle KLM \sim \triangle \underline{QRS}$$

Scale Factor:  $\frac{6}{4} = \frac{3}{2}$



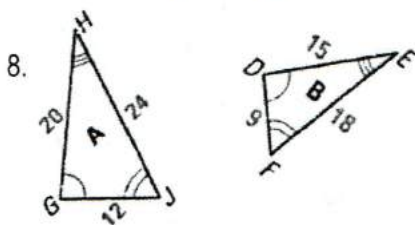
$$CBAD \sim \underline{YXWZ}$$

Scale Factor:  $\frac{3}{4}$



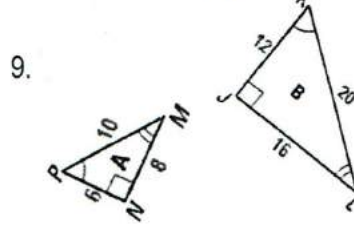
$$RSPQ \sim \underline{HJFG}$$

Scale Factor:  $\frac{4}{10} = \frac{2}{5}$



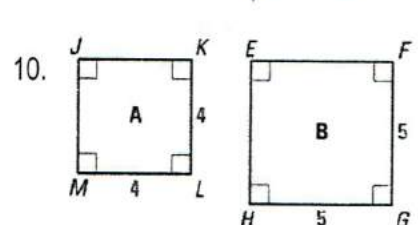
$$\triangle HJG \sim \triangle \underline{EFD}$$

Scale Factor:  $\frac{20}{15}$  or  $\frac{24}{18}$  or  $\frac{12}{9}$   $\boxed{\frac{4}{3}}$



$$\triangle NPM \sim \triangle \underline{JKL}$$

Scale Factor:  $\frac{6}{12}$  or  $\frac{8}{16}$  or  $\frac{10}{20}$   $\boxed{\frac{1}{2}}$

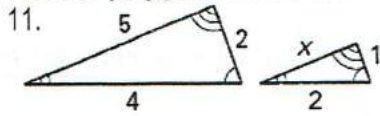


$$KJML \sim \underline{FEHG}$$

Scale Factor:  $\frac{4}{5}$



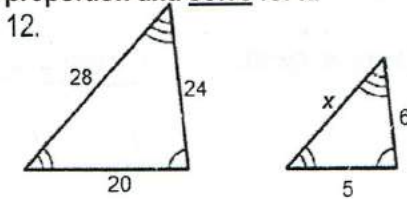
The two polygons are similar. Write a proportion and solve for x.



$$\frac{5}{x} = \frac{4}{2}$$

$$\frac{4x}{4} = \frac{10}{4}$$

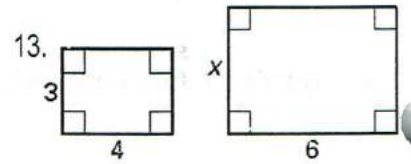
$$x = \frac{5}{2}$$



$$\frac{28}{x} = \frac{20}{5}$$

$$20x = 140$$

$$x = 7$$

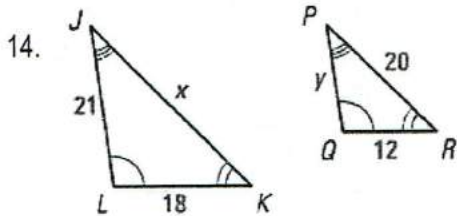


$$\frac{3}{x} = \frac{4}{6}$$

$$4x = 18$$

$$x = \frac{18}{4} = \frac{9}{2}$$

Complete the similarity statement for the similar figures and then find the scale factor.  
Next, write proportions and SOLVE for the missing lengths.



$$\triangle JLK \sim \triangle PQR$$

$$\frac{x}{20} = \frac{18}{12}$$

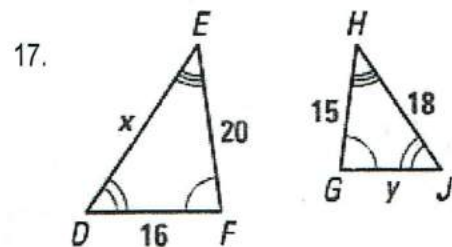
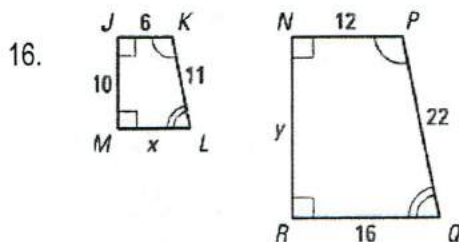
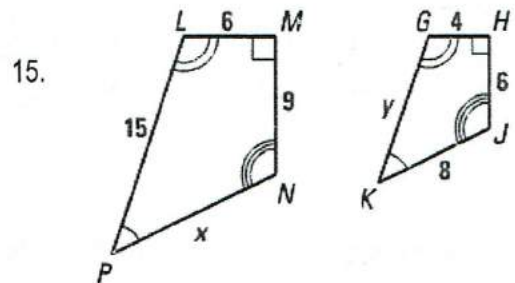
$$12x = 360$$

$$x = 30$$

$$\frac{21}{y} = \frac{18}{12}$$

$$18y = 252$$

$$y = 14$$



$$\frac{x}{18} = \frac{20}{15}$$

$$15x = 360$$

$$x = 24$$

$$\frac{16}{y} = \frac{20}{15}$$

$$20y = 240$$

$$y = 12$$

Similar Figures

Each pair of figures is similar. Find the missing side.

1)

$$\frac{12}{3} = \frac{20}{x} \quad \boxed{x=5}$$

$$12x = 60$$

2)

$$\frac{x}{9} = \frac{1}{3} \quad \boxed{x=3}$$

$$3x = 9$$

3)

$$\frac{x}{8} = \frac{4}{16}$$

$$16x = 32$$

$$\boxed{x=2}$$

4)

$$\frac{5}{x} = \frac{4}{8}$$

$$40 = 4x$$

$$\boxed{x=10}$$

5)

$$\frac{14}{2} = \frac{x}{1}$$

$$14 = 2x$$

$$\boxed{x=7}$$

6)

$$\frac{9}{x} = \frac{6}{24}$$

$$6x = 216$$

$$\boxed{x=36}$$

7)

$\triangle ABC \sim \triangle FED$

$$\frac{9}{99} = \frac{10}{x}$$

$$9x = 990$$

$$\boxed{x=110}$$

8)

$$\frac{10}{x} = \frac{10}{100}$$

$$10x = 1000$$

$$\boxed{x=100}$$