

Congruence and Triangles

Essential question: What can you conclude about two triangles that are congruent?

When you know that two triangles are congruent, you can make conclusions about the sides and angles of the triangles.

COMMON CORE

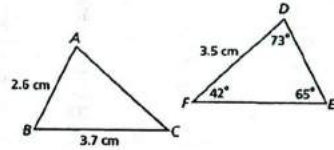
CC.9-12.G.CO.7

1 EXAMPLE Finding an Unknown Dimension

$\triangle ABC \cong \triangle DEF$. Find DE and $m\angle B$. Explain your reasoning.

A. Complete the following to find DE .

Because $\triangle ABC \cong \triangle DEF$, there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$.



This same sequence of rigid motions maps \overline{AB} to \overline{DE} .

This means $\overline{AB} \cong \overline{DE}$.

Congruent segments have the same length, so $AB = DE$.

$AB = 2.6 \text{ cm}$, so $DE = 2.6 \text{ cm}$

B. To find $m\angle B$, use similar reasoning to show that $\angle B \cong \angle E$.

So, $m\angle B = 65^\circ$

REFLECT

1a. If you know $\triangle ABC \cong \triangle DEF$, what six congruence statements about segments and angles can you write? Why?

$\overline{AB} \cong \overline{DE}$ $\angle A \cong \angle D$
 $\overline{BC} \cong \overline{EF}$ $\angle B \cong \angle E$
 $\overline{AC} \cong \overline{DF}$ $\angle C \cong \angle F$

When two triangles are congruent, the corresponding parts are the sides and angles that are images of each other. You write a congruence statement for two figures by matching the corresponding parts. In other words, the statement $\triangle ABC \cong \triangle DEF$ contains the information that \overline{AB} corresponds to \overline{DE} (and $\overline{AB} \cong \overline{DE}$), $\angle A$ corresponds to $\angle D$ (and $\angle A \cong \angle D$), and so on.

The following theorem is often abbreviated CPCTC. The proof of the theorem is similar to the argument presented in the previous example.

Corresponding Parts of Congruent Triangles are Congruent Theorem (CPCTC)

If two triangles are congruent, then corresponding sides are congruent and corresponding angles are congruent.

The converse of CPCTC is also true. That is, if you are given two triangles and you know that the six pairs of corresponding sides and corresponding angles are congruent, then you can conclude that the triangles are congruent. In the next lesson, you will see that you need only three pairs of congruent corresponding parts in order to conclude that the triangles are congruent, provided they are chosen in the right way.

Corresponding Parts of Congruent Triangles are Congruent

CPCTC

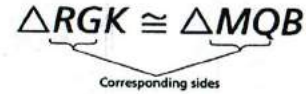
2 EXAMPLE Using CPCTC

$\triangle RGK \cong \triangle MQB$. Write six congruence statements about corresponding parts.

A. Identify corresponding sides.

Corresponding sides are pairs of letters in the same position on either side of the congruence statement.

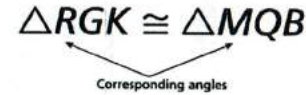
$\overline{RG} \cong \overline{MQ}$, $\overline{GK} \cong \overline{QB}$, $\overline{RK} \cong \overline{MB}$



B. Identify corresponding angles.

Corresponding angles are letters in the same position on either side of the congruence statement.

$\angle R \cong \angle M$; $\angle G \cong \angle Q$; $\angle K \cong \angle B$



REFLECT

2a. Given that $\triangle PQR \cong \triangle STU$, $PQ = 2.7$ ft, and $PR = 3.4$ ft, is it possible to determine the length of \overline{TU} ? If so, find the length. If not, explain why not.

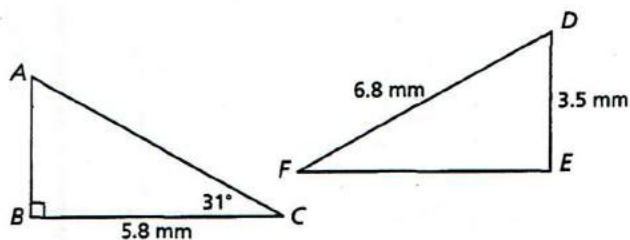
no - $\overline{TU} \cong \overline{QR}$, but we don't know QR .

2b. A student claims that any two congruent triangles must have the same perimeter. Do you agree or disagree? Why?

H

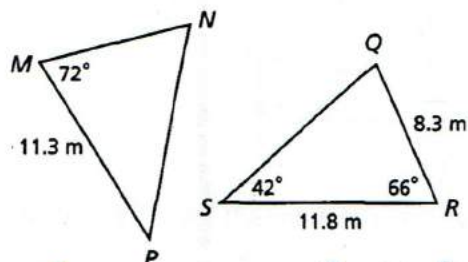
PRACTICE

1. $\triangle ABC \cong \triangle DEF$. Find AB and $m\angle E$.



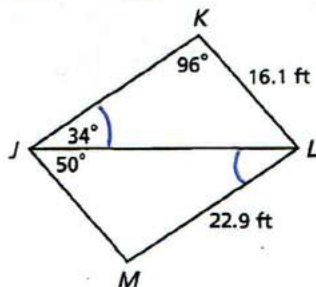
$AB = 3.5 \text{ mm}, m\angle E = 90^\circ$

2. $\triangle MNP \cong \triangle QRS$. Find NP and $m\angle P$.



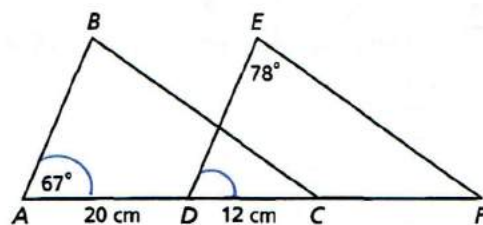
$NP = 11.8 \text{ m}, m\angle P = 42^\circ$

3. $\triangle JKL \cong \triangle LMJ$. Find JK and $m\angle JLM$.



$JK = 22.9 \text{ ft}, m\angle JLM = 34^\circ$

4. $\triangle ABC \cong \triangle DEF$. Find DF and $m\angle EDC$.



$DF = 32 \text{ cm}, m\angle EDC = 67^\circ$

For each given congruence statement, write six congruence statements about corresponding parts.

5. $\triangle JWT \cong \triangle GKH$

$\overline{JW} \cong \overline{GK}$ $\angle J \cong \angle G$
 $\overline{WT} \cong \overline{KH}$ $\angle W \cong \angle K$
 $\overline{JT} \cong \overline{GH}$ $\angle T \cong \angle H$

6. $\triangle PQL \cong \triangle KYU$

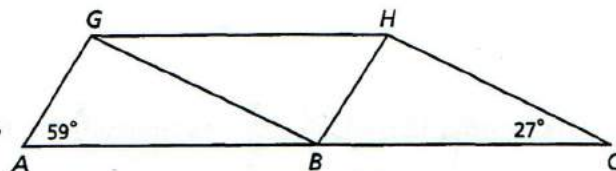
$\overline{PQ} \cong \overline{KY}$ $\angle P \cong \angle K$
 $\overline{QL} \cong \overline{YU}$ $\angle Q \cong \angle Y$
 $\overline{PL} \cong \overline{KU}$ $\angle L \cong \angle U$

7. $\triangle HTJ \cong \triangle NRZ$

$\overline{HT} \cong \overline{NR}$
 $\overline{TJ} \cong \overline{RZ}$
 $\overline{HTJ} \cong \overline{NRZ}$

8. The figure shows a portion of the truss of a bridge. $\triangle ABG \cong \triangle BCH \cong \triangle HGB$.

a. Is it possible to determine $m\angle GBH$? If so, how? If not, why not?



b. A student claims that B is the midpoint of \overline{AC} . Do you agree? Explain.

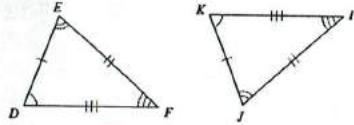
Congruence and Triangles

Name _____

Date _____ Period _____

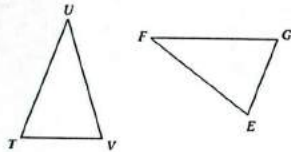
Complete each congruence statement by naming the corresponding angle or side.

1) $\triangle DEF \cong \triangle KJI$



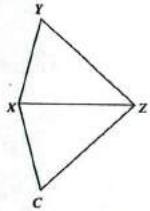
$\overline{FD} \cong ? \overline{IK}$

3) $\triangle TUV \cong \triangle GFE$



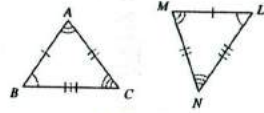
$\angle U \cong ? \angle F$

5) $\triangle ZXY \cong \triangle ZXC$



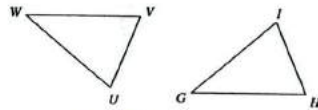
$\angle Y \cong ? \angle C$

2) $\triangle BAC \cong \triangle LMN$



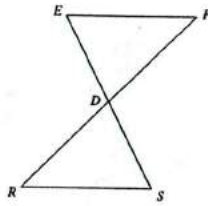
$\angle A \cong ? \angle M$

4) $\triangle WVU \cong \triangle GHI$



$\angle W \cong ? \angle G$

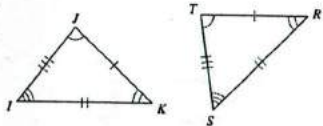
6) $\triangle DEF \cong \triangle DSR$



$\angle F \cong ? \angle R$

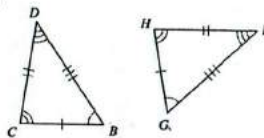
Write a statement that indicates that the triangles in each pair are congruent.

7)



$\triangle JKI \cong \triangle TRS$

8)



$\triangle BCD \cong \triangle GHI$

or

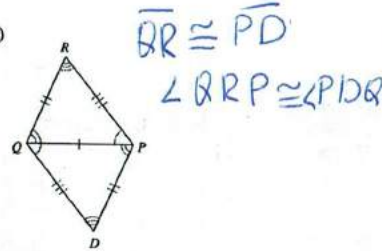
$\triangle CDB \cong \triangle HIG$

or

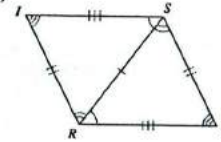
$\triangle DCB \cong \triangle IHG$

-1-

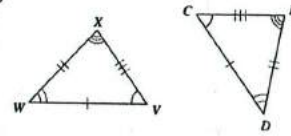
9)



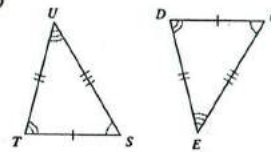
10)



11)

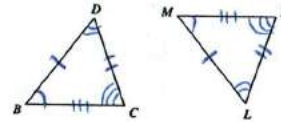


12)

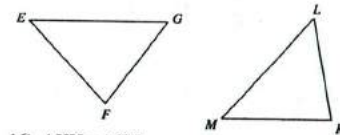


Mark the angles and sides of each pair of triangles to indicate that they are congruent.

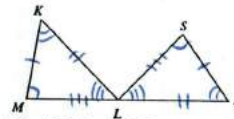
13) $\triangle BDC \cong \triangle MLK$



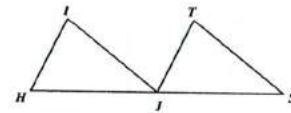
14) $\triangle GFE \cong \triangle LKM$



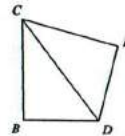
15) $\triangle MKL \cong \triangle STL$



16) $\triangle HIJ \cong \triangle JTS$



17) $\triangle CDB \cong \triangle CDL$



18) $\triangle IJK \cong \triangle JCD$



-2-