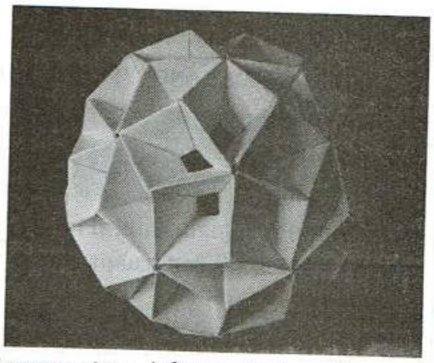
Lesson 5 Symmetries of Quadrilaterals



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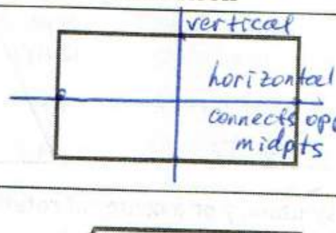
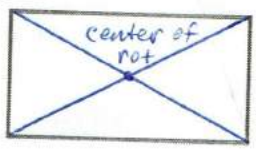
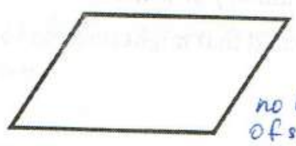
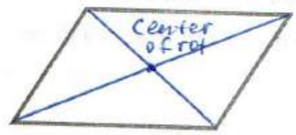
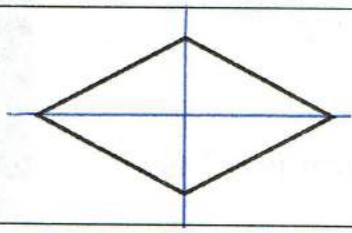
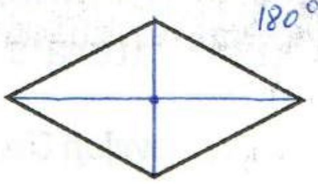
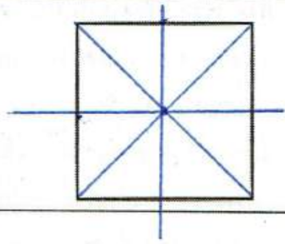
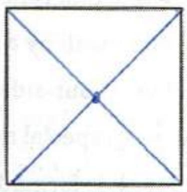
A Develop Understanding Task

A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**.

Every four-sided polygon is a **quadrilateral**. Some quadrilaterals have additional properties and are given special names like squares, parallelograms and rhombuses. A **diagonal** of a quadrilateral is formed when opposite vertices are connected by a line segment. Some quadrilaterals are symmetric about their diagonals. Some are symmetric about other lines. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

For each of the following quadrilaterals you are going to try to answer the question, "Is it possible to reflect or rotate this quadrilateral onto itself?" As you experiment with each quadrilateral, record your findings in the following chart. Be as specific as possible with your descriptions.

Defining features of the quadrilateral	Lines of symmetry that reflect the quadrilateral onto itself	Center and angles of rotation that carry the quadrilateral onto itself
A **rectangle** is a quadrilateral that contains four right angles.		 180° rot. symmetry
A **parallelogram** is a quadrilateral in which opposite sides are parallel.	 no lines of symmetry	

A **rhombus** is a quadrilateral in which all sides are congruent.		180° 
A **square** is both a rectangle and a rhombus.		90° 

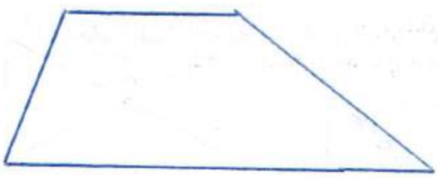
A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. Is it possible to reflect or rotate a trapezoid onto itself?

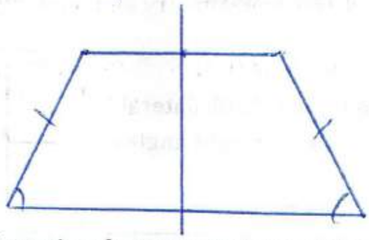
Draw a trapezoid based on this definition. Then see if you can find:

- any lines of symmetry, or
- any centers of rotational symmetry,

that will carry the trapezoid you drew onto itself.

isosceles trapezoid





If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid that might possess some type of symmetry.

Lesson 5

READY, SET, GO! Name Period Date

READY

Topic: Polygons, definition and names

1. What is a polygon? Describe in your own words what a polygon is.

A polygon is a closed shape with straight sides.
The word polygon comes from the Greeks and means many angles.

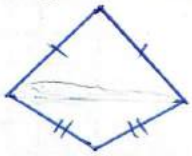
2. Fill in the names of each polygon based on the number of sides the polygon has.

Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

SET

Topic: Kites, Lines of symmetry and diagonals.

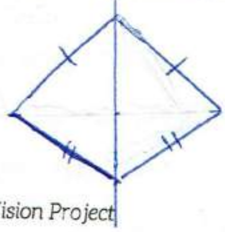
3. One quadrilateral with special attributes is a kite. Find the geometric definition of a kite and write it below along with a sketch. (You can do this fairly quickly by doing a search online.)



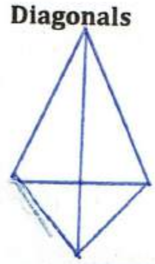
Kite is a qudrilateral with two distinct pairs of adjacent congruent sides.

4. Draw a kite and draw all of the lines of reflective symmetry and all of the diagonals.

Lines of Reflective Symmetry



Diagonals



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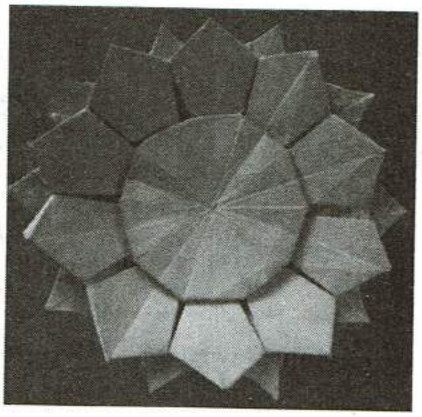
Lesson 6 Symmetries of Regular Polygons

A Solidify Understanding Task



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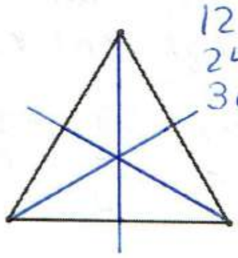
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

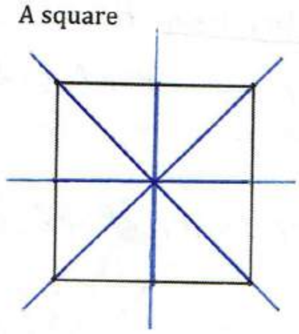
For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

1. An equilateral triangle



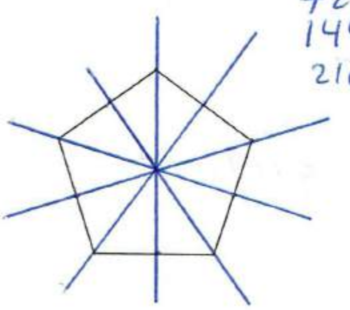
120°
240°
360°

2. A square



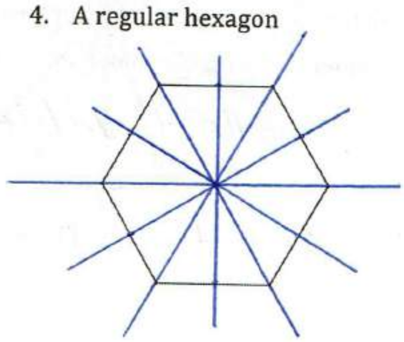
90°
180°
270°
360°

3. A regular pentagon



72°
144°
216°
288°
360°

4. A regular hexagon

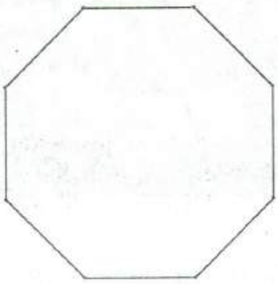


60°
120°
180°
240°
300°
360°

5. A regular octagon

8 lines of symmetry

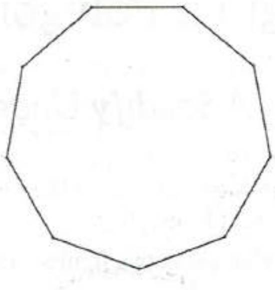
45°
90°
135°
180°
225°
270°
315°
360°



6. A regular nonagon

9 lines of symmetry

40°
80°
120°
160°
200°
240°
280°
320°
360°



What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon? # of lines of symmetry = # of sides = # of vertices

For odd # of sides: vertex to midpt of opp. side (passes through center)

For even # of sides: vertex to opp. vertex or midpt to opp. midpt } (pass through center)

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

Smallest value is $\frac{360^\circ}{\text{\# of sides}}$

and then multiples of that quotient

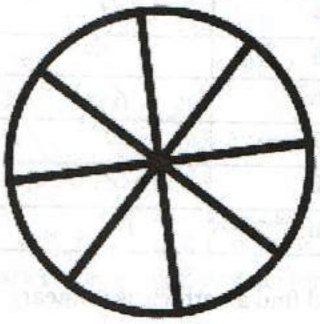
Lesson 6

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Rotational symmetry, connected to fractions of a turn and degrees.

1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



$\frac{1}{8}$

45°

2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?

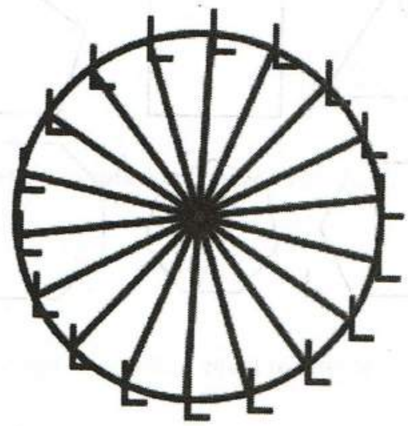
$\frac{1}{5}$

72°

3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?

$\frac{1}{18}$

20°



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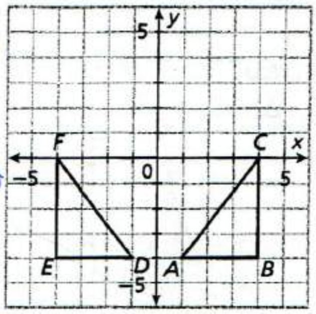
PRACTICE

Use the definition of congruence in terms of rigid motions to determine whether the two figures are congruent and explain your answer.

Reflection over y-axis

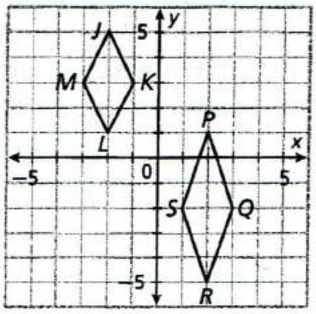
$\triangle FED \cong \triangle CBA$

1.



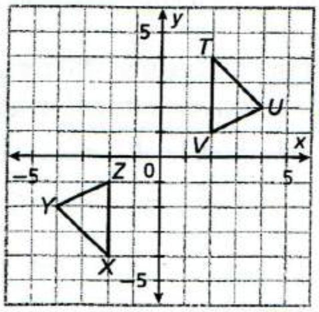
Reflections are rigid motions and rigid motions produce ≅ figures.

2.



no – they are not ≅ b/c you cannot get there by rigid motions.

3.



Rotation of 180°

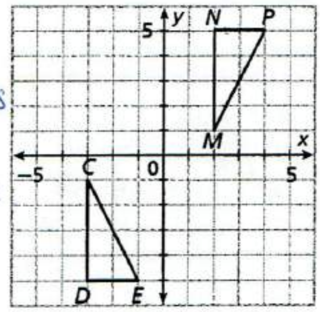
$\triangle TUV \cong \triangle XYZ$

Rotations are rigid motions and rigid motions produce ≅ figures.

For each pair of congruent figures, find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

Reflect over x-axis then translate left 5

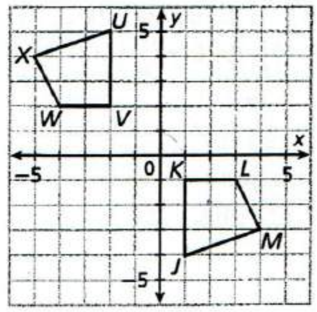
4.



$T(x,y) \to (x+5, y)$

Reflections and translations are rigid motions, and rigid motions produce ≅ figures

5.

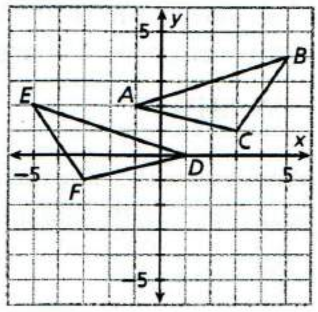


Rot 180°

$T(x,y) \to (x-1, y+1)$

UVWX to JKLM

6.



Ref over y-axis

$T(x,y) \to (x, y+2)$

$\triangle EFD$ to $\triangle BCA$

7. $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle GHJ$. Can you conclude $\triangle ABC \cong \triangle GHJ$? Explain.

yes – by transitive prop. of ≅

$\triangle$s are ≅ if you can get from one to another by a rigid motion or by a sequence of rigid motions.

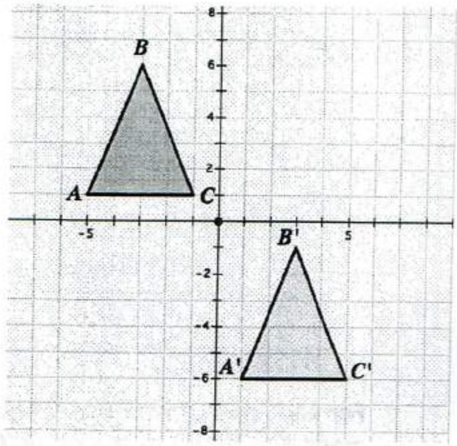
Congruent Triangles and Rigid Motion Name ____________________

Directions: The following problems deal with congruency and rigid motion. The term "rigid motion" is also known as "isometry" or "congruence transformations."

1. In the diagram at the right, a transformation has occurred on $\triangle ABC$.

   a) Describe a transformation that created image $\triangle A'B'C'$ from $\triangle ABC$.

   translation
   $(x, y) \rightarrow (x+6, y-7)$



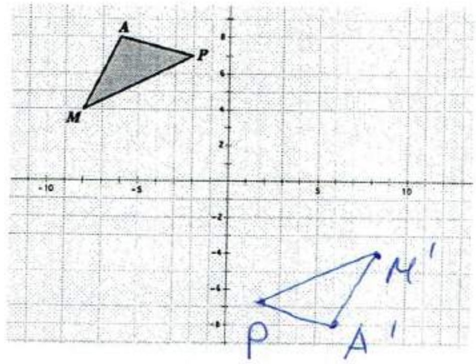
   b) Is $\triangle ABC$ congruent to $\triangle A'B'C'$? yes Explain.

   translations are rigid motions and rigid motions produce congruent figures.

2. The vertices of $\triangle MAP$ are M(-8, 4), A(-6, 8) and P(-2, 7).
   The vertices of $\triangle M'A'P'$ are M'(8, -4), A'(6, -8) and P'(2, -7).

   a) Plot $\triangle M'A'P'$.

   b) Verify that the sides of the triangles are congruent using the distance formula.



   c) Describe a rigid motion that can be used to obtain $\triangle M'A'P'$

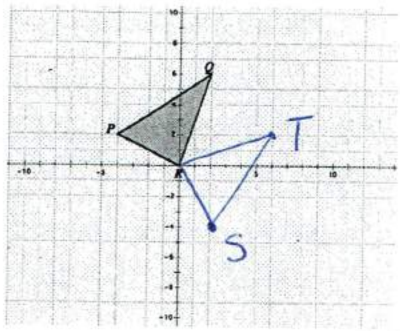
   180° rot

3. Given $\triangle PQR$ with P(-4, 2), Q(2, 6) and R(0, 0) is congruent to $\triangle STR$ with S(2, -4), T(6, 2) and R(0, 0).

   a) Plot $\triangle STR$.

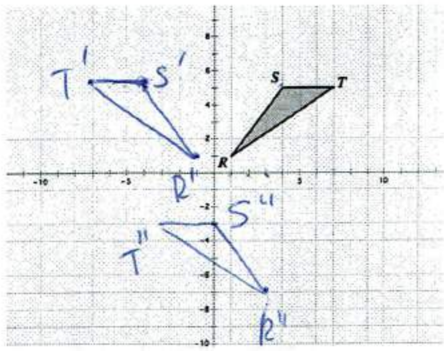
   b) Describe a rigid motion which can be used to verify the triangles are congruent.

   ref. over y=x axis



2

4. Given ΔRST with R(1, 1), S(4, 5) and T(7, 5).



a) Plot the reflection of ΔRST in the y-axis and label it $\Delta R'S'T'$.

b) Is ΔRST congruent to $\Delta R'S'T'$? yes Explain.

ref. are rigid motion and rigid motions produce ≅ figures

c) Plot the image of $\Delta R'S'T'$ under the translation $(x, y) \rightarrow (x + 4,\ y - 8)$. Label the image of $\Delta R''S''T''$.

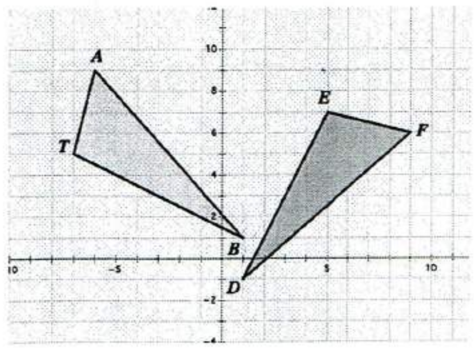
d) Is $\Delta R'S'T'$ congruent to $\Delta R''S''T''$? yes Explain.

translations are rigid motions and rigid motions produce ≅ figures

e) Is ΔRST congruent to $\Delta R''S''T''$? yes Explain.

ref + translation

5. Given ΔDFE with D(1, -1), F(9, 6) and E(5,7) and ΔBAT with B(1, 1), A(-6, 9) and T(-7, 5).

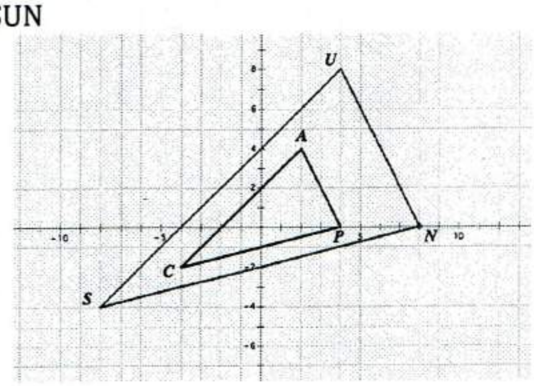


a) Describe a transformation that will yield ΔBAT as the image of ΔDFE.

rot. 90° CCW

b) Is ΔBAT congruent to ΔDFE? yes Explain.

6. Given ΔCAP with C(-4, -2), A(2, 4) and P(4, 0) and ΔSUN with S(-8, -4), U(4, 8) and N(8, 0).



a) Describe a transformation that will yield ΔSUN as the image of ΔCAP.

dilation with magnitude 2

b) Is ΔCAP congruent to ΔSUN? no Explain.

dilation is not a rigid motion